Paper – I

B.Sc I Integral Calculus

Notes 4

Type III. When the denominator contains quadratic factors such as $a_1x^2 + b_1x + c_1$, $a_2x^2 + b_2x + c_2$ etc then the given fraction can be written as

$$\frac{Ax+B}{a_1x^2+b_1x+c_1} + \frac{Cx+D}{a_1x^2+b_2x+c_2} + \ldots$$

where A, B, C, D ..... are constants to be determined and then the integration can be performed

Example. $\int \frac{dx}{x^3-1}$

Solⁿ Let $\frac{1}{x^3-1}$ i.e. $\frac{1}{(x-1)(x^2+x+1)}$

$$= \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$$

or, $1 = A(x^2+x+1) + (Bx+C)(x-1)$

Equating the coefficients of x^2, x, x^0 successively, we get

$$A + B = 0 \quad — (1)$$

$$A - B + C = 0 \quad (2)$$

$$A - C = 1 \quad (3)$$

From (1) & (3) $A = -B$ & $A = C + 1$

$$\Rightarrow \quad -B = C + 1$$

Putting these value in (2)

$$C + 1 + C + 1 + C = 0$$

$$3C = -2$$

$$C = -2/3$$

$$\Rightarrow B = -(C+1) = -\left(-\frac{2}{3} + 1\right)$$

$$= -\left(\frac{-2+3}{3}\right) = -1/3$$

$$A = -2/3 + 1 = \frac{-2+3}{3} = 1/3$$

Now $I = \frac{1}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{x+2}{x^2+x+1}\,dx$

$= \frac{1}{3}\log(x-1) - \frac{1}{3}\cdot\frac{1}{2}\int \frac{2x+1+3}{x^2+x+1}\,dx$

$= \frac{1}{3}\log(x-1) - \frac{1}{6}\int \frac{2x+1}{x^2+x+1}\,dx - \frac{1}{2}\int \frac{dx}{(x+\frac{1}{3})^2+\frac{3}{4}}$

$= \frac{1}{3}\log(x-1) - \frac{1}{6}\log(x^2+x+1) - \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{(x+\frac{1}{3})}{\sqrt{3}/2}$

$+ C$

Type C

When the denominator contains repeated quadratic factors such as $(a_1x^2+b_1x+c_1)^3$, etc then the given fraction can be expressed as

$\frac{Ax+B}{a_1x^2+b_1x+c} + \frac{Cx+D}{(a_1x^2+b_1x+c_1)^2} + \frac{Ex+F}{(a_1x^2+b_1x+}$

$+ \text{-----------}$. where A, B, C, D $E, F,$ ---- are constants to be evaluated and finally the integration

Can be carried out.

Examples

$$\int \frac{dx}{\sin x + \sin 2x}$$

Here $I = \int \frac{dx}{\sin x + 2\sin x \cos x}$

$$= \int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x \, dx}{\sin^2 x(1+2\cos x)}$$

Put $\cos x = t$ $\therefore -\sin x \, dx = dt$

$$\therefore I = -\int \frac{dt}{(1-t^2)(1+2t)}$$

Now $\frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$

Putting $t = 1, -1, -1/2$ successively we get

$A = 1/6$ $B = -1/3$, $C = 4/3$

$$\therefore\ I = -\tfrac{1}{6}\int \frac{dt}{1-t} + \frac{1}{2}\int \frac{dt}{1+t} - \frac{4}{3}\int \frac{dt}{1+2t}$$

$$= -\frac{1}{6}\log(1-t) + \frac{1}{2}\log(1+t) - \frac{4}{3}\log(1+2t) + C$$